MSc in Bioinformatics for Health Sciences
MAT. Elements of Mathematics

Syllabus Information

Academic Course: 2019/20

Academic Center: 804 - Official Postgraduate Programme in Biomedicine

Study: 8045 – Bioinformatics for Health Sciences - MSc

Subject: 31035 – MAT. Elements of Mathematics

Credits: 5.0

Course: 1st

Teaching languages: English

Teachers: Ferran Muiños

Teaching Period: 1st term

Presentation

The unreasonable effectiveness of mathematics is one of the most remarkable feats of human endeavor. Through history, mathematical thinkers have come up with ideas that make science significantly much simpler and more beautiful. However, it requires from beginners an initial investment of energy and, most critically, getting rid of all prejudices.

The goal of this course is to provide the students with a minimal toolkit and understanding of the ways researchers in computational biology model nature and solve practical problems.

This course will include a minimum of theory classes, alongside hands-on scientific computing sessions and written exercises. The course will require the students to carry on practice in the form of self-study and interaction with their peers, enough to develop good intuitions about the tools that we will explore.

A minimum proof of work will be necessary to access the final exam. In particular, a journal club will be conducted in which students will explain to their peers how a specific mathematical application works.

Associated skills

General competences:

Instrumental:

1. Read-write-listen English related to widely used math concepts and techniques.
2. Discuss and select appropriate ways to solve a particular problem.
3. Basic calculations in algebra and calculus – either with pen-paper or computer.
4. Understanding methods widely applied in computational biology.
5. Scientific computing within a read-eval-print loop.

Interpersonal:

1. Make oneself clear, concise and specific for scientific discussions.
2. Contribute to discussions to solve a problem.

Systemic:

1. Analysis, abstraction and synthesis.
2. Understand formal mathematical language and proofs.
3. Understand the limits of own understanding.
4. Search for information.

Specific competences:

1. Enumeration of subsets, selections and relations.
2. Combinatorial designs.
3. Vectors and linear maps. Generators, basis and change of coordinates.
4. Matrices as linear maps. Determinants. Isomorphisms.
5. Diagonalization of matrices.
6. Scalar product, metrics, norms, orthogonality, angles.
7. Functions of one and several variables.
8. Continuous functions.
9. Derivative of a function. Chain rule. Smooth functions.
10. Gradient. Jacobian and Hessian matrices.
11. Convex functions. Local and global optimization.
12. Gradient descent. Back-propagation.
13. Lagrange multipliers method for optimization subject to equality constraints.
14. Kuhn-Tucker method for optimization subject to inequality constraints.

Contents

Block 1: Grounds for a new language

Topic	Concepts
Getting used to the language	Sets, Functions; Spaces, Morphisms; Sequences; Recurrence; Graph of a Function.
Perspective of common applications in computational biology	Counting Paths in a Network; Clustering; Phylogenetic reconstruction; Markov Chains; Page-Rank algorithms; Linear regression; Principal Component Analysis; Signal deconstruction; Feed-forward Neural Networks.

Block 2: Basic discrete mathematics

Topic	Concepts
Enumeration	Subsets, Selections, Permutations, Combinations (Pascal), Partitions (Stirling).
Graphs	Graphs, Bipartite graphs, Trees, Directed Graphs.

Block 3: Linear Algebra

Topic	Concepts
Vector spaces and subspaces	Specification of vector spaces and subspaces; Homogeneous systems of linear equations.
Linear dependence, generators, basis	Basis of a vector space; Change of basis.
Linear maps	Linear maps; Isomorphisms; Associated matrix in a basis; Change of coordinates; Kernel of a linear map.
Diagonalization of matrices, eigenvectors and eigenvalues	Eigenvalues and eigenvectors of a linear map; Diagonalizable matrices; Singular Value Decomposition (SVD); Principal Component Analysis (PCA).

Block 4: Optimization

Topic	Concepts
Basic topology	Open, closed and compact sets.
Continuous functions	Domain of a function where it is continuous.
Smooth functions	Derivatives; Partial derivatives; Gradient; Tangent and normal space.
Criticality	Critical points of a function; local and global extrema; saddle points.
Optimization	Optimization with equality constraints (Lagrange Multipliers). Optimization with inequality constraints (Kuhn-Tucker conditions). Gradient descent; Back-propagation.

Teaching methods

- In the classroom (30 hours): Work in the classroom includes the correct understanding of the theory and practical exercises. Moreover, a more practical work applying the theory to easy problem solving and examples is expected from the student.

- Outside the classroom (95 hours).: Activities outside the classroom includes the resolution of problem sets (one for each section of the course) and the complete understanding the mathematical proofs studied in class

Evaluation

General assessment criteria:

The evaluation will consist on a final exam at the end of the course, worth 50%; a problem set delivered during the course, worth 30%; a journal club presentation, worth 20%.

Grading system

Grades are between 0 and 10 and an overall 5 is needed to pass.

Bibliography:

- *Concrete Mathematics: A Foundation for Computer Science.* Ronald L. Graham, Donald E. Knuth, Oren Patashnik. Addison-Wesley, 1994.
- *The Art of Computer Programming (Volume 1): Fundamental Algorithms.* Donald E. Knuth. Addison-Wesley, 1968.
- *3Brown1Blue* [YouTube Channel]. Grant Sanderson. URL: https://www.youtube.com/c/3blue1brown
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- *Introduction to Linear Algebra.* Gilbert Strang. URL: http://math.mit.edu/~gs/linearalgebra
- *Linear Algebra and its Applications.* David C. Lay, Steven R. Lay, Judi J. McDonald. Pearson, 2016.
- *A First Course in Calculus.* Serge Lang. Springer, 1986.
- *Elementary Classical Analysis.* Jerrold E. Marsden, Michael J. Hoffman. W. H. Freeman, 1993.
- *Infinite Powers: How Calculus Reveals the Secrets of the Universe.* Steven Strogatz. Houghton Mifflin Harcourt, 2019.